Population dynamics in spatially heterogeneous systems with drift: The generalized contact process

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We investigate the time evolution and stationary states of a stochastic, spatially discrete, population model (contact process) with spatial heterogeneity and imposed drift (wind) in one and two dimensions. We consider in particular a situation in which space is divided into two regions: an oasis and a desert (low and high death rates). Carrying out computer simulations we find that the population in the (quasi) stationary state will be zero, localized, or delocalized, depending on the values of the drift and other parameters. The phase diagram is similar to that obtained by Nelson and coworkers from a deterministic, spatially continuous model of a bacterial population undergoing convection in a heterogeneous medium.

I. INTRODUCTION

In a series of works David Nelson and collaborators [1,2] investigated the extinction versus survival of bacterial colonies undergoing convection in a medium with heterogeneous distribution of nutrients. Space is divided into an "oasis," with high nutrients, and a "desert" with low nutrients. The above model is similar to models used in mathematical epidemiology in which a population is divided into subpopulations (patches) and disease-causing factors are nonuniformly distributed across patches [3].

Nelson and collaborators [1,2], solved the Kolmogorov-Fisher equation (KFE) with a convective term, using its similarity to a nonlinear Schrödinger equation with a non-Hermitian perturbation. The eigenvalue spectrum of this equation has a diverging localization length with a sharp mobility edge. This mobility edge is identified with the Fisher velocity, the speed of propagation in the KFE[4]. This means that while at high convection velocity all bacteria are transported into the desert and become extinct, at low velocity the surviving population is localized near the oasis. If the desert can support a nonzero steady state population extinction is replaced by a delocalized steady state.

It is tempting to consider the continuum deterministic equations of Nelson, [1,2] as a coarse grained description of a more microscopic stochastic model of this system [5–13]. A potential candidate for such a model is a generalized contact process (CP) on a lattice which includes both drift and spatial heterogeneity. Using Monte Carlo simulations, we find that this stochastic model does in fact have a phase diagram which is qualitatively similar to that obtained from Nelson's deterministic continuum approach. Various transitions between phases (extinct, localized, and delocalized) are identified.

Stochastic modeling of the discrete microscopic space-time events underlying the type of physical phenomena considered in [1–3] raises many conceptual and methodological questions. These are further complicated by the practical limitations of computer simulations—the size of the system, the time it can be followed, and the number of samples are all strictly limited and are generally small compared to the macroscopic space-time scales of the physical phenomena. Experience nevertheless indicates that judicious microscopic modeling, which requires choosing system sizes and simulation times large enough to well represent "typical" behavior plus "normal" fluctuations, can add to our understanding of the macroscopic phenomena [5–7,11]. On the other hand the simulation time must not be too large so as to avoid rare events such as extinction of populations, which will always occur eventually in finite systems. These, however, occur on a time scale that grows exponentially with the size of the system and so will almost never be seen at intermediate time scales. We do not attempt here any deep analysis of these important issues but refer to the literature [5,10,11].

The more modest goal of this paper is to investigate the effect of locomotion with drift, in an inhomogeneous spatial environment, on the behavior of the stochastic contact process. This is a frequently used model for population dynamics, spread of epidemics, etc. [3–5,7,11,12]. We consider in particular an environment consisting of a single oasis surrounded by a patch of desert several times the size of the oasis. By using periodic boundary conditions, this is equivalent to considering a periodic array of such patches which, quite unrealistically, are microscopically identical. We first describe the basic contact process with additional spatial jumps and find that these jumps have the effect of destroying correlations. This brings the system closer to its mean field behavior. We then investigate the oasis-desert system in one and two dimensions. Hydrodynamical scaling which connects this stochastic model to continuum deterministic models is discussed briefly.

II. DESCRIPTION OF MODEL

The contact process (CP) is frequently used to model population dynamics where the population lives on the d-dimensional cubic lattice Z^d [5,7,10–12]. At each site x of the lattice there is an organism which can exist in one of two states $\{0,1\}$: We will refer to 0 as dead or 1 as alive. Let $\eta_x = 0, 1$ be the variable which specifies the state at a site x of a d-dimensional simple cubic lattice Z^d and let $\underline{\eta} = \{\eta_y\}$, $y \in Z^d$, be the configuration of the system at some given time. The stochastic dynamics of the process are specified by the following rates:

$$\eta_x = 1 \rightarrow \eta_x = 0 \text{ at rate } \alpha(x), \tag{1}$$

$$\eta_x = 0 \rightarrow \eta_x = 1 \text{ at rate } (\lambda/2d) \sum_{y:|y-x|=1} \eta_y, \tag{2}$$

where $2d$ is the number of neighbors of site x. Equivalently, a live site dies at rate $\alpha(x)$ or attempts to give birth to a new bacterium on a randomly chosen neighboring site at rate $\lambda/2d$. It succeeds if that site is empty, otherwise nothing happens. We are assuming here for simplicity that the birth rate is spatially uniform, although of course one might expect both to depend on local conditions. In particular one expects higher birth rates in the oasis than in the desert. What is most important, however, is just the ratio of birth rate to death rate, hence the effect we are modeling should be essentially captured by just considering variations in the latter.

As there must be at least one live site to propagate, the all-zero state is absorbing, i.e., if $\eta_x = 0$ for all x at any time t it will stay like that forever [5,11,12]. When $\alpha(x) = \alpha$ is constant, the evolution depends only on the ratio λ/α. For large λ/α a live population can survive. As λ/α decreases there is a transition from survival to extinction at a critical value of this ratio which depends on the dimension d of the lattice. In units of $\alpha = 1$, $\lambda_c \simeq 3.30$ in one dimension and $\lambda_c \simeq 1.65$ on the square lattice [5,11].

In the mean-field (MF) approximation, corresponding to the neglect of correlations between different sites, one can write a deterministic equation for the density of live sites. Thus, for the spatially homogeneous case, with density $\langle \eta_x(t) \rangle = \rho(t)$, we have

$$\frac{d\rho(t)}{dt} = -\alpha\rho + \lambda(1-\rho)\rho^2. \tag{3}$$

Starting with some nonzero density $\rho(0) \leq 1$ the long time solution of Eq. (3) is $\rho(\infty) = 0$ if $\alpha/\lambda > 1$, and $\rho(\infty) = 1 - \alpha/\lambda$, if $\alpha/\lambda < 1$, and so $\lambda_c^{(MF)} = 1$. This shows in particular that $\lambda_c^{(MF)} < \lambda_c$ for $d = 1, 2$ (or indeed for any finite d). This can be understood by noting that correlations between live (occupied) sites are positive for the contact process in the stationary state, i.e.: $\langle \eta_x \eta_y \rangle \geq \langle \eta_x \rangle \langle \eta_y \rangle$. Thus correlations inhibit growth, since live sites give birth only on unoccupied neighboring sites.

We now add to the CP a mechanism for the movement of the bacteria via a Kawasaki exchange kinetics [5,7–12]. This permits a bacterium at site x to move (jump) to a neighboring site y, $|x-y| = 1$. The exchanges conserve the total number of individuals in the system. These two processes can be considered as follows. A clock at each live site x rings at a rate 1, i.e., the probability that it will ring in a very small time interval dt is equal to dt. When it rings the bacterium at that site attempts to jump to a neighboring site with probability P_K, dies with probability $(1-P_K)\alpha(x)/c$, or attempts to give birth to a bacterium on a randomly chosen neighboring site with probability $(1-P_K)\lambda/(2dc)$, and with probability $(1-P_K)[1-(\alpha(x)+\lambda/2d)/c]$ nothing happens. The constant c is chosen as small as possible to make all probabilities nonnegative, i.e., $c = \lambda/2d + \max_x \alpha(x)$. We note further that if the bacteria decides to jump or give birth on a neighboring site which is already occupied, nothing happens.

If a jump is decided upon, the appropriate neighboring site is chosen as follows: In one dimension a bacterium at site x attempts to jump to the site $x+1$ with probability q and to the site $x-1$ with probability $1-q, 0 \leq q \leq 1$. In two dimensions the symmetry of the Kawasaki exchanges along the y axis is broken while the exchanges along the x axis are symmetric. A bacterium at site (x,y) attempts to jump to site $(x,y+1)$ with probability $q/2$ and to site $(x,y-1)$ with probability $(1-q)/2$ while it tries to jump to site $(x+1,y)$ and to site $(x-1,y)$ with equal probability $1/4$. When $q > 1/2$ there is a drift to the right or up (left or down for $q < 1/2$). In all cases jumps succeed if the target site is empty, otherwise nothing happens.

When $q = 1/2$, particles jump in each direction with equal probability. This corresponds to a symmetric stirring. For constant α and $q = 1/2$ it has been shown in Ref. [8] that in the limit $P_K \rightarrow 1$, one can rescale space and time appropriately to obtain rigorously a continuum reaction-diffusion equation for the density. A space-time scaling, leading to a reaction-diffusion equation with drift holds also for $q > \frac{1}{2}$; see Sec. VI. This means that when the jumps become very frequent they destroy the correlations between sites. Subsequently the critical value of λ assumes its mean-field value in this limit.

In general the configuration probability distribution $\mu(\underline{\eta}, t)$ evolves in time according to the master equation

$$\frac{\partial \mu(\underline{\eta}, t)}{\partial t} = [P_K L_K + (1 - P_K) L_G] \mu(\underline{\eta}, t), \tag{4}$$

where L_G describes the Glauber dynamics, changing η_x to $1 - \eta_x$, while L_K describes the Kawasaki exchanges between neighboring sites $|x-y| = 1$ [5,8].

III. ONE-DIMENSIONAL SYSTEM

A. Homogeneous system

We carried out Monte Carlo simulations of the contact process with a homogeneous death rate $\alpha = 1$ and with symmetric Kawasaki exchanges for various jump probabilities P_K. The system was a ring containing 50, 100, 200, 300, 500 sites (with periodic boundary conditions). Results were obtained by taking averages over $10^3 - 10^4$ realizations, each starting with an initial configuration obtained by putting a bacterium at each site with probability ρ_0. For each realization ρ_0 was chosen uniformly in the interval $(0,1)$. As is well
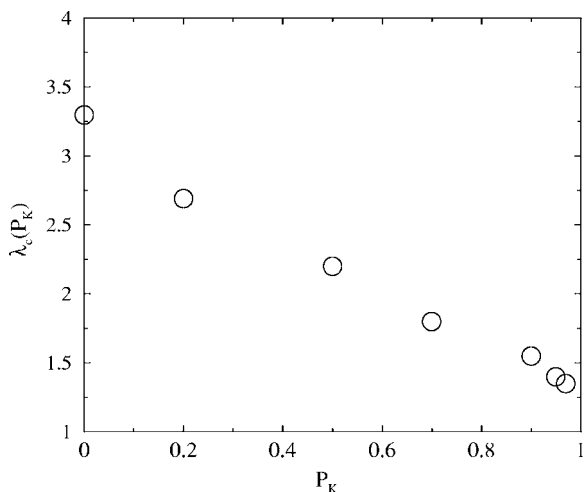
FIG. 1. Critical values $\lambda_c(P_K)$ as a function of the jump probability P_K for $q=0.5$. Results are extrapolated to $N=\infty$ (open circles) from system sizes of $N=50, 100, 200, 300, 500$. The average is taken over 10^3–10^4 different realizations starting with random initial configurations at density.

known the only steady state in the finite system is the absorbing state corresponding to $\eta_x=0$ for all x [5,11]. It is only in an infinite system where there can be steady states with a nonvanishing population density. It is therefore necessary to extrapolate the critical values of the infinite system from the results of the simulations of finite systems [5]. The extrapolated critical values of the CP in one dimension are shown in Fig. 1 as a function of P_K. The simulations indicate an approximately linear decrease of λ_c with P_K and are consistent with the expectations that stirring reduces correlations. As already noted in the limit of rapid stirring the system behaves in a mean-field way with $\lambda_c(P_K)\rightarrow 1$ as $P_K \rightarrow 1$ [8].

We also simulated the homogeneous contact process with drift in one dimension. The critical values λ_c, for $P_K=0.9$ are plotted in Fig. 2 for different values of q. We find that the critical value λ_c decreases approximately linearly as q increases to 1; here again the drift helps survival by reducing correlations between occupied sites. It does this by randomly exposing new empty sites on which births can take place. Such an effect does not occur for a deterministic ballistic motion which satisfies Gallilean invariance.

B. Spatial heterogeneity

We introduce two types of spatially varying death rates in one dimension

$$\alpha_I(x) = 1 + \epsilon \sin\left(\frac{2\pi x}{N}\right) \qquad (5)$$

$$\alpha_{II}(x) = \begin{cases} 1-\epsilon, & x \in \text{ Oasis} \\ 1+\epsilon, & x \in \text{ Desert} \end{cases}, \qquad (6)$$

where $x \in \{1,2,\dots,N\}$ with periodic boundary condition $0 < \epsilon < 1$. The oasis is located in $\{x|1 \leqslant x \leqslant N/5\}$ and the desert is everywhere else. For $\epsilon=0$, $\alpha=1$ sets the unit of time.
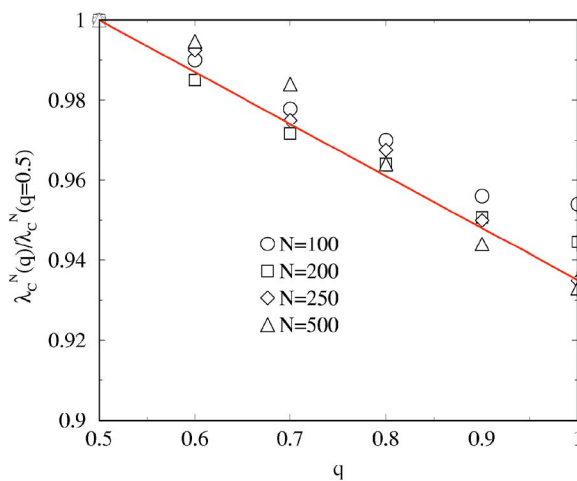

FIG. 2. (Color online) Critical value of $\lambda_C^N(q)$ for a homogeneous system as a function of drift q in one dimension. $P_K=0.9$ and system sizes of $N=100, 200, 250, 500$ are used. The function that is used for the solid line is $1-\gamma(q-0.5)$ with $\gamma=0.13$.

For both types of spatial variation we used simulations to obtain the critical values $\lambda_c(\epsilon)$, defined as the smallest value of λ for which a population can survive as a function of ϵ. In the absence of exchanges, $P_K=0$, the critical value $\lambda_c(\epsilon)$ decreases monotonically as ϵ increases. The ratios of $\lambda_c(\epsilon)/\lambda_c(0)$ are plotted against ϵ in Fig. 3. We find

$$\lambda_c(\epsilon)/\lambda_c(0) \simeq 1 - \epsilon = \text{Min}_x \alpha(x). \qquad (7)$$

Equation (7) suggests that the system behaves, for large N, as consisting of approximately independent local regions (patches) with different values of the death rate $\alpha(x)$. If that were true exactly we would have $\lambda_c(\epsilon)/\lambda_c(0)=1-\epsilon$. The stationary population density $\bar{\rho}(x)$ is plotted in Fig. 4 for both heterogeneous and homogeneous death rates using a value of
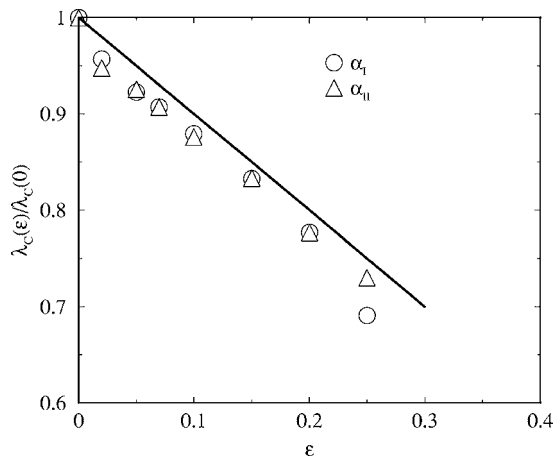

FIG. 3. Ratio of the critical values in the heterogeneous system to those in the homogeneous system, $\lambda_c(\epsilon)/\lambda_c(0)$. The parameters used for the simulation are $P_K=0$ and the results are extrapolated to $N=\infty$ (open symbols) from $N=50, 100, 200, 500, 1000$. The average is taken over 10^3-10^4 different realizations. The function that is used for the solid line is $1-\epsilon$.
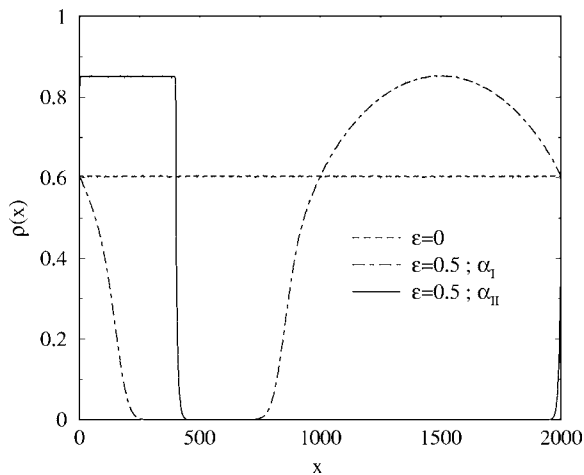
FIG. 4. Stationary density $\bar{\rho}(x)$ on an one dimensional ring of size $N=2000$ for $P_K=0$ and $\lambda=4$. $\alpha_I(x)$ and $\alpha_{II}(x)$ with $\epsilon=0.5$ are used, respectively.

λ which is smaller than λ_c for $\alpha=1+\epsilon$ but larger than λ_c for $\alpha=1-\epsilon$.

C. Spatial heterogeneity with exchanges

In this section we investigate the steady states of the contact process with both Kawasaki exchange kinetics and a spatial heterogeneity of the form $\alpha_{II}(x)$ given in Eq. (6). The nature of the stationary states of the system now depend on the strength of the drift q.

In the simulations we chose $P_K=0.9$ so that we could see the change of the stationary states as a response to the variation of the drift, while keeping the total simulation time, which should increase [8] proportionally to $1/(1-P_K)$ manageable. The critical values $\lambda_c(q)$ were obtained for different values of the drift q in systems of size, $N=50, 100, 200, 500$, and then were extrapolated to those in an infinite system.

The λ-q phase diagram is shown in Fig. 5 for $\epsilon=0.5$. The critical curve (solid line), $\lambda_c(q)$ separates the survival and extinction regions. For $\lambda \leq \lambda_c(q)$ the extinct phase $\eta_x=0$ for all x is the only steady state. For $\lambda > \lambda_c(q)$ there is a steady state with a finite population density which will be approached as $t \rightarrow \infty$, starting with a nonzero initial density. There are two survival steady states: a localized and a delocalized one. In the localized survival phase the population density is nonzero only in the oasis and its vicinity while in the delocalized survival phase the population density is nonzero everywhere. As ϵ decreases the region of the localized phase in the phase diagram shrinks. In the limit $\epsilon=0$ the critical curve collapses onto the slanted critical curve of the contact process with drift and homogeneous death rate $\alpha=1$ [see Fig. 2].

The dependence of the stationary density profile $\rho(x)$ on q and λ are shown in Figs. 6 and 7. Figure 6 shows how the population in the oasis is decreased as the wind (drift q) increases at $\lambda=1.5$. For this value of λ the oasis is able to sustain a population while the desert is not: $\lambda_{c2}(\epsilon) < \lambda < \lambda_{c1}(\epsilon)$ where $\lambda_{c1}(\epsilon)$ [or $\lambda_{c2}(\epsilon)$] is defined as the critical value for the CP with symmetric Kawasaki exchange with
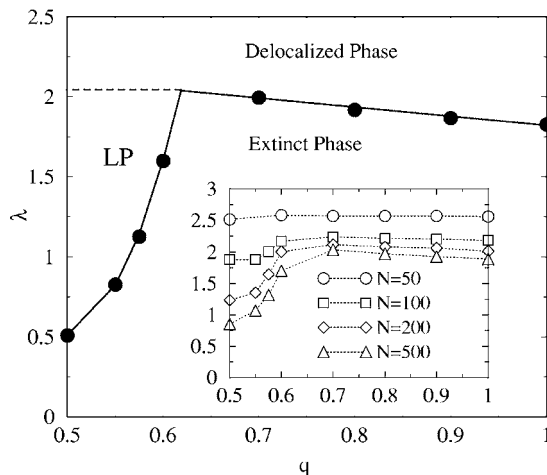

FIG. 5. λ-q phase diagram of the contact process with drift and spatial heterogeneity, $\alpha_{II}(x)$ with $\epsilon=0.5$, in one dimension. Here $P_K=0.9$. The critical curve (solid line with filled circles), obtained by extrapolation to the infinite system from $N=50, 100, 200, 500$ (shown in the inset), separates the survival (delocalized or localized) from extinct phase. LP stands for the localized phase.

$P_K=0.9$ and homogeneous death rate $\alpha=1+\epsilon$ [or $\alpha=1-\epsilon$]. There is a critical drift, $q_c(\lambda)$ for each fixed λ, above which the population becomes extinct in the long time limit. When the drift is weak, $[q<q_c(\lambda)]$ the surviving population is localized near the oasis. When the drift is strong, $[q>q_c(\lambda)]$ the population is "blown" into the desert and becomes extinct. We see from Fig. 5 (where $\epsilon=0.5$), that $\lambda_{c1}(\epsilon=0.5) \simeq 2$, $\lambda_{c2}(\epsilon=0.5) \simeq 0.5$, and $q_c(\lambda_{c1}) \simeq 0.6(1)$.

Figure 7 shows the density profile for a strong drift $q=0.9$ at different λ's. The condition for survival, when the drift is stronger than the critical drift, is that the desert should be able to sustain a population, i.e., $\lambda > \lambda_{c1}(\epsilon)$. For $\lambda > \lambda_{c1}(\epsilon)$ the population survives everywhere and its steady state is the delocalized phase. The condition for this survival
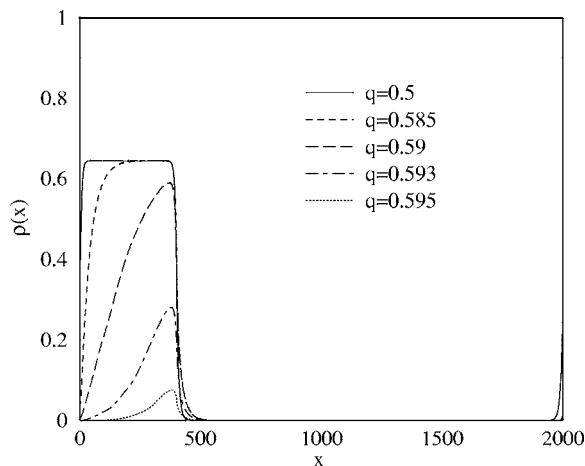

FIG. 6. Drift-driven transition from the localized phase to the extinct phase. The parameters used for the simulation are $N=2000$, $\lambda=1.5$, $\alpha_{II}(x)$ with $\epsilon=0.5$, and $P_K=0.9$. Drift varies from $q=0.5$ where population is localized in the vicinity of the oasis to $q=0.598$ where the population becomes extinct.
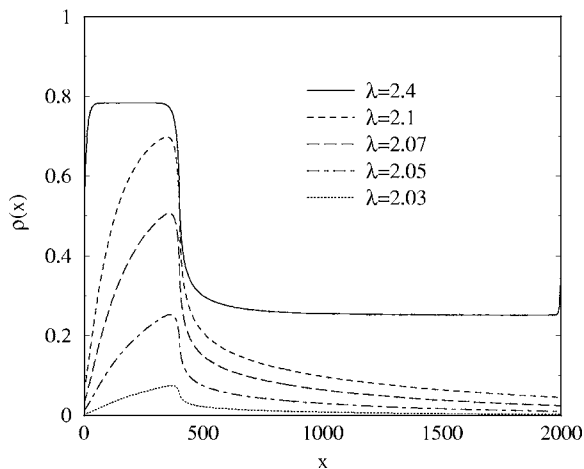
FIG. 7. Transition from the delocalized survival phase to the extinct phase. The parameters used for the simulation are system size of $N=2000$, $q=0.9$, $\alpha_{II}(x)$ with $\epsilon=0.5$, and $P_K=0.9$. Birth rate varies from $\lambda=2.4$ where population is extended everywhere to $\lambda=1.9$ where the population becomes extinct.

is very much similar to that of the CP with a homogeneous death rate of $\alpha=1+\epsilon$ (desert) and with strong drift ($q=0.9$).

When $q < q_c(\lambda)$, a transition from the localized to the delocalized survival phase can be achieved by making λ cross λ_{c1}, the dashed line in the λ-q phase diagram in Fig. 5. When λ is very close to λ_{c1}, it is also possible to have a transition from the localized phase (through the extinction phase) to the delocalized phase for constant λ. This is due to the slanted critical curve in the λ-q plane: One can draw a horizontal line crossing three phases in the λ-q plane: the population is localized at small drift (undergoes an extinction transition,) and becomes delocalized at strong drift.

IV. SPATIAL HETEROGENEITY WITH EXCHANGES IN TWO DIMENSIONS

In $d=2$ the exchange kinetics impose a drift along the y direction. We consider a periodic box of length N and choose the region of oasis to be $\{(x,y)|2N/5 < x \leqslant 3N/5, 2N/5 < y \leqslant 3N/5\}$ and the desert everywhere else. We considered square lattices of sizes $N^2=30^2, 50^2, 70^2, 100^2$, with periodic boundary conditions, for $\epsilon=0.5$ in Eq. (6) and $P_K=0.9$, and extrapolated the critical curve to that in an infinite system.

Note that the steady states in two dimensions also depends on the shape of the oasis. An oasis of narrow width along the direction of the drift will yield a very different steady state from one having a narrow width perpendicular to the direction of the drift. One can expect that in the former case the steady states will be similar to those of the CP with heterogeneous death rate $\alpha_{II}(x)$ with symmetric exchanges in one dimension while for the latter case the steady states will be similar to those of the CP with $\alpha_{II}(x)$ under the drift in one dimension.

The λ-q phase diagram for the CP with drift and with spatial heterogeneity in two dimensions is plotted in Fig. 8 for various system sizes with the oasis a square with sides of length $N/5$. The localized [delocalized] phase in two dimen-
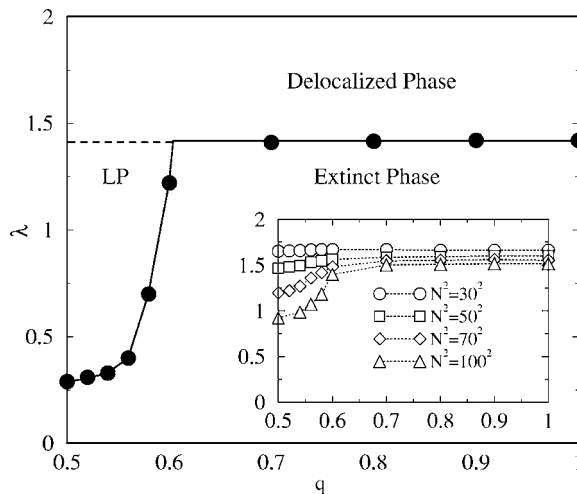

FIG. 8. λ-q phase diagram of the contact process with drift and spatial heterogeneity in two dimension. The parameters used for the simulation are $P_K=0.9$, $\alpha_{II}(x)$ with $\epsilon=0.5$. The critical curve (solid line with filled circles) is obtained by extrapolation to the infinite system from $N^2=30^2, 50^2, 70^2, 100^2$ (shown in the inset). The oasis is a square of length $N/5$. LP stands for the localized phase.

sions is identified by the presence of a large cluster of occupied sites around the oasis, which is "not large enough" ["large enough"] to reach both boundaries of the system along the x or y direction. The delocalized, or localized survival phase and the extinction phase are separated by the critical curve $\lambda_c(q)$. The shape of that critical curve is similar to that in one dimension. This is not too surprising since, unlike equilibrium systems, nonequilibrium systems like the CP, can have phase transitions also in one dimension with similar phase diagrams [5]. We can therefore expect similar phase transitions between three phases by varying (λ, q): Examples of localized and delocalized phases are depicted in Figs. 9 and 10.

V. TIME EVOLUTION OF THE POPULATION DENSITY $\rho_t(x)$

To study the time evolution of the population density $\rho_t(x)$ undergoing drift in a heterogeneous system, we performed
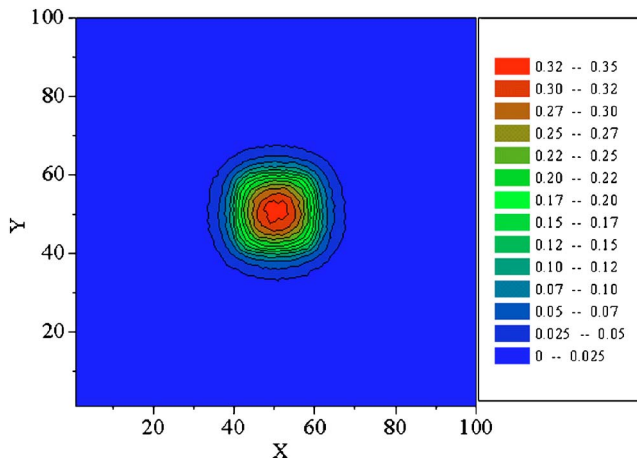

FIG. 9. (Color online) Localized phase in two dimension for $\lambda=1.05$, $q=0.5$, $\alpha_{II}(x)$ with $\epsilon=0.5$, $P_K=0.9$, and $N^2=100^2$.
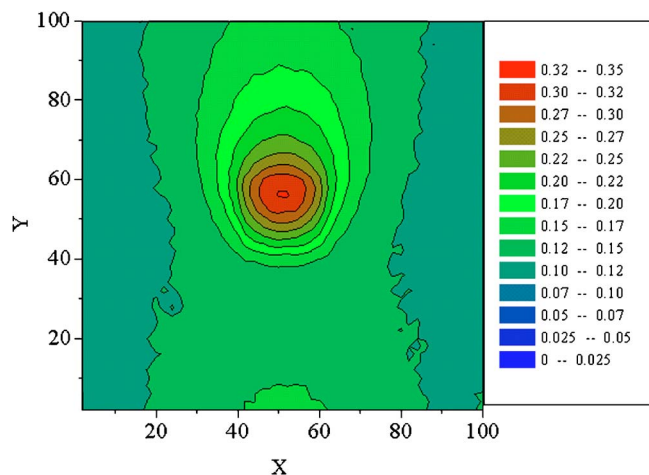
FIG. 10. (Color online) Delocalized phase in two dimension for $\lambda=1.75$, $q=0.7$, $\alpha_{II}(x)$ with $\epsilon=0.5$, $P_K=0.9$, and $N^2=100^2$.

Monte Carlo simulations in one dimension with $N=2000$, $P_K=0.9$, and $\epsilon=0.5$ in Eq. (6). We selected a set of (λ, q) parameters from different steady states in the λ-q phase diagram in Fig. 5 and evolved the system from various initial conditions.

In the localized and in the delocalized phases we started with a population density $\rho_0=0.01$ randomly placed. For each set of parameters we simulated 10^4 realizations of the process with the above initial conditions. After 100, 200, 500, 2000 Monte Carlo time steps we measured the population density $\rho_t(x)$. (In one Monte Carlo time step each site is visited once for updating its state, on average.)

Figures 11 and 12 show the time evolution of a population density under a weak drift ($q=0.59$) for a small birth rate ($\lambda=1.5$) and for a large birth rate ($\lambda=2.5$). For the small birth rate ($\lambda=1.5$) the population grows mostly in the vicinity of the oasis and its density profile is skewed to the right due to the presence of the drift: Under a stronger drift the population will die out in this case. For the large birth rate
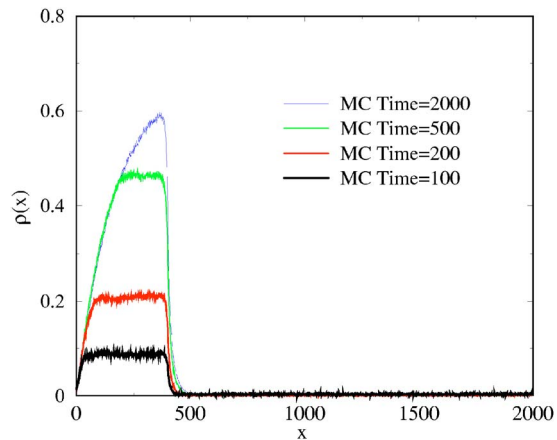

FIG. 11. (Color online) Time evolution of population density $\rho_t(x)$ for $\lambda=1.5$, $q=0.59$, $\alpha_{II}(x)$ with $\epsilon=0.5$, and $N=2000$, i.e., for a weak drift in the localized phase. 10^4 realizations starting with 1% of randomly distributed population are averaged at given Monte Carlo time steps.
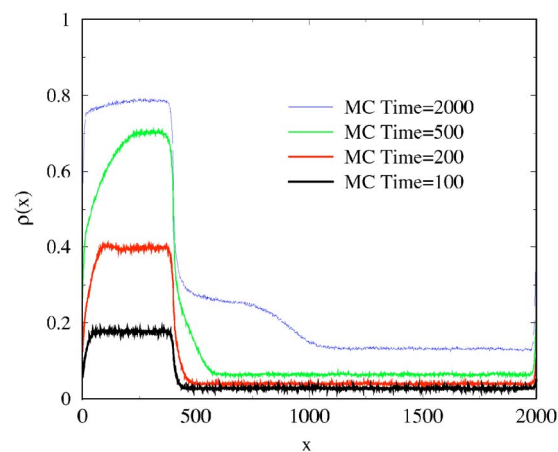

FIG. 12. (Color online) Time evolution of population density $\rho_t(x)$ for $\lambda=2.5$, $q=0.59$, $\alpha_{II}(x)$ with $\epsilon=0.5$, $P_K=0.9$, and $N=2000$, i.e., for a weak drift in the delocalized phase. 10^4 realizations starting with 1% of randomly distributed population are averaged at given Monte Carlo time steps.

($\lambda=2.5$) the population density profile at 2000 Monte Carlo time steps decreases gradually in the desert as one goes further away from the oasis. Note that in the strong drift case, $q=0.9$ shown in Fig. 7, the population density profile is flatter in the desert for $\lambda=2.4$.

VI. COMPARISON OF THE CONTACT PROCESS AND THE KOLMOGOROV-FISHER EQUATION

De Masi *et al.* [8] derived a reaction diffusion equation for the symmetric exclusion process with added Glauber dynamics by rescaling time and space, $x=\epsilon ja$ ($j=,\ldots,$ $-1,0,1,\ldots,$) and $t=\epsilon^2\tau, a$ and τ being the microscopic lattice spacing and τ being the time in microscopic units (jump time) and then taking the $\epsilon\to 0$ limit with $(1-P_K)\sim\epsilon^2$.

We expect that under suitable scalings the model considered in this paper leads, when $P_K\to 1$, to a continuum macroscopic equation of the form [8,14]

$$\frac{\partial P_t(x)}{\partial t} = D\nabla^2 P_t(x) - v\nabla P_t(x)[1-P_t(x)] + [\lambda-\alpha(x)]P_t(x)$$

$$- \lambda P_t^2(x), \tag{8}$$

where $P_t(x)$ is the bacteria density at time t at position x, D is a diffusion constant, and v is a convection velocity where D and v can be obtained from the microscopic model. (Note that on the time and space scales considered here there is no contribution to D from the birth and death proven, i.e., $D \to 0$ when $P_K\to 0$.)

Equation (8) resembles the continuum modified KFE used by Nelson [1,2], which generalizes the KFE to account for convection and an inhomogeneous growth rate

$$\frac{\partial c(x,t)}{\partial t} = D_N\nabla^2 c(x,t) - v_N\nabla c(x,t) + U(x)c(x,t) - bc^2(x,t),$$

$$(9)$$

where $c(x,t)$ is the bacteria population density, D_N is the (Nelson) diffusion constant of the bacteria, $U(x)$ is the spa-

tially varying growth potential corresponding to $[\lambda - \alpha(x)]$ in Eq. (8), v_N is an externally imposed (Nelson) convection, and b is a parameter that limits the population number density to a maximum saturation value. The only difference between Eqs. (8) and (9) is the nonlinear convection term in Eq. (8). Because our interest lies mainly in the location of the critical curve, we can linearize Eq. (8), while keeping a nonlinear reaction term. Then the coefficients in the two equations can be matched with each other. This makes it possible to compare the simulation results with those from the differential equation. We find good qualitative agreement between two approaches, even though we do not present the results of comparison here.

VII. CONCLUSIONS

We studied the steady states and dynamical behavior of the contact process with asymmetric Kawasaki exchange kinetics and with spatially varying death rate. This is a stochastic, spatially discrete, version of the deterministic continuum Kolmogorov-Fisher equation with convection velocity, investigated by Nelson and collaborators [1,2]. The phase diagrams from both the stochastic and the deterministic cases, birth rate λ versus q, are similar: Two (localized and delocalized) survival phases are separated from the extinction phase by a critical curve, $\lambda_c(q)$. As is true in many other nonequilibrium situations, the difference between one and two dimension is small. Simulations of the time evolution of the population under drift in one-dimensional heterogeneous systems show how a population evolves to the localized or the delocalized steady states, depending on the birth rate.

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